Math 215(4.5&4.6) Supp to Hw 10 Name & Number:

1. Two species, X and Y, live in a symbiotic relationship. I.e. neither species can survive on its own and each depends on the other for its survival. Initially, there are 15 of X and 10 of Y. If x = x(t) and y = y(t) are the sizes of the populations at time t months, the growth rates of the two populations are given by the system

$$x' = -0.8x + 0.4y$$
$$y' = 0.4x - 0.2y$$

Determine what happens to these two populations.

2. Solve the system.

(a) $\begin{aligned} x' &= x + 3y,\ x(0) = 0 \\ y' &= 2x + 2y,\ y(0) = 5 \end{aligned}$

(b) $\begin{aligned} x_1' &= x_1 + x_2, \\ x_2' &= x_1 - x_2 \end{aligned}$

3. An automobile rental company has three locations, which we designate as P, Q, and R. When an automobile is rented at one of the locations, it may be returned to any of the three locations. Suppose, at some specific time, that there are p cars at location P, q cars at Q, r cars at R. Experience has shown, in any given week, that the p cars at location P are distributed as follows: 10% are rented and returned to Q, 30% are rented and returned to R, and 60% remain at P(these either are not rented or are rented and returned to P). Similar rental histories are known for locations Q and R, as summarized below.

Weekly distribution history

Location P: 60 % stay at P, 10% go to Q, 30% go to R

Location Q: 10% go to P, 80% stay at Q, 10% go to R

Location R: 10% go to P, 20% go to Q, 70% stay at R

Let $\vec{x}_k = \begin{pmatrix} p(k) \\ q(k) \\ r(k) \end{pmatrix}$ represent the state of the rental fleet at the beginning of week k, where $p(k), q(k)$ and $r(k)$ are numbers at each locations P, Q and R.

Suppose there are 200 cars initially at each place.

(a) Determine the number of cars at each location in the first week, fifth week and 10^{th} week.

(b) Find the steady state vector. i.e. $\lim_{n \to 0} x_n$. Describe your answer in words.